

CBCS SCHEME

USN

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21MAT41

Fourth Semester B.E. Degree Examination, June/July 2023 Complex Analysis, Probability and Statistical Methods

Time: 3 hrs.

Max. Marks: 100

*Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Use of Statistical table is permitted.*

Module-1

- 1 a. Derive Cauchy – Riemann equations in Cartesian form. (06 Marks)
- b. Show that $f(z) = \sin z$ is analytic and hence find its derivative. (07 Marks)
- c. Evaluate $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3y - x)dy$, along the parabola $x = 2t, y = t^2 + 3$ (07 Marks)

OR

- 2 a. Determine the analytic function $f(z) = u + iv$, whose imaginary part is $(x^2 - y^2) + \frac{x}{x^2 + y^2}$ by Milne – Thompson method. (06 Marks)
- b. State and prove Cauchy's integral theorem. (07 Marks)
- c. Evaluate $\int_c \frac{dz}{z^2 - 4}$ over $c: |z| = 1$ (07 Marks)

Module-2

- 3 a. Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ (06 Marks)
- b. If α and β are the two roots of $J_n(x) = 0$ then prove that $\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = 0$ if $\alpha \neq \beta$. (07 Marks)
- c. Express $f(x) = 2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. (07 Marks)

OR

- 4 a. Obtain the series solution of Bessel's differential equation $x^2 y'' + xy' + (x^2 + n^2)y = 0$ leading to $J_n(x)$. (06 Marks)
- b. Show that $J_{+\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ (07 Marks)
- c. Prove that, $x^3 + 2x^2 - 4x + 5 = \frac{2}{5}P_3(x) + \frac{4}{3}P_2(x) - \frac{17}{5}P_1(x) + \frac{17}{5}P_0(x)$ (07 Marks)

Module-3

- 5 a. Find the Karl Pearson's coefficient correlation for the following two groups.

| | | | | | | | | | | |
|---|----|----|----|----|----|----|----|----|----|----|
| A | 92 | 89 | 87 | 86 | 83 | 77 | 71 | 63 | 53 | 50 |
| B | 86 | 83 | 91 | 77 | 68 | 85 | 52 | 82 | 37 | 57 |

(06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- b. Fit a straight line of the form $y = ax + b$ for the data by the least squares method.

| | | | | | | |
|---|---|---|----|----|----|----|
| x | 0 | 1 | 2 | 3 | 4 | 5 |
| y | 9 | 8 | 24 | 28 | 26 | 20 |

(07 Marks)

- c. Using the method of least squares fit a curve $y = ax^b$ for the data

| | | | | | |
|---|-----|---|-----|---|------|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 0.5 | 2 | 4.5 | 8 | 12.5 |

(07 Marks)

OR

- 6 a. Ten students got the percentage of marks in two subjects x and y. Compute their rank correlation coefficient.

| | | | | | | | | | | |
|------------|----|----|----|----|----|----|----|----|----|----|
| Marks in x | 78 | 36 | 98 | 25 | 75 | 82 | 90 | 62 | 65 | 39 |
| Marks in y | 84 | 51 | 91 | 60 | 68 | 62 | 86 | 58 | 53 | 37 |

(07 Marks)

- b. Compute the means \bar{x} , \bar{y} and the coefficient of correlation r from the given regression lines $2x + 3y + 1 = 0$, $x + 6y - 4 = 0$. (07 Marks)
- c. Fit a second degree parabola $y = ax^2 + bx + c$ in the least square sense for the following data and hence estimate y at $x = 6$.

| | | | | | |
|---|----|----|----|----|----|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 10 | 12 | 13 | 16 | 19 |

(06 Marks)

Module-4

- 7 a. A random variable X has the following probability function :

| | | | | | | | | |
|------|---|---|----|----|----|-------|--------|------------|
| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| P(X) | 0 | k | 2k | 2k | 3k | k^2 | $2k^2$ | $7k^2 + k$ |

Find k and evaluate $P(X \geq 6)$, $P(3 < X \leq 6)$.

(06 Marks)

- b. Find the mean and standard deviation of Poisson distribution. (07 Marks)
- c. The probability that a person aged 60 years will live upto 70 is 0.65. What is the probability that out of 10 persons aged 60 atleast 7 of them will live upto 70? (07 Marks)

OR

- 8 a. Find a constant K such that

$$f(x) = \begin{cases} kx^2, & 0 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases} \text{ is a pdf.}$$

Also, compute : (i) $P(1 < x < 2)$ (ii) $P(x \leq 1)$ (iii) $P(x > 1)$

(06 Marks)

- b. Find the mean and standard deviation of Binomial distribution. (07 Marks)
- c. In a test of electric bulbs it was found that the lifetime of bulbs of a particular brand was normally distributed with an average life of 2000 hours and standard deviation of 60 hours. If a firm purchases 2500 bulbs, find the number of bulbs that are likely to last for
- More than 2100 hours
 - Less than 1950 hours
 - Between 1900 and 2100 hours

Given that, $\phi(1.67) = 0.4525$; $\phi(0.83) = 0.2967$

(07 Marks)

Module-5

- 9 a. The joint probability distribution of the random variables X and Y are given as follows:

| | | | |
|-------|---------------|----------------|----------------|
| X \ Y | 1 | 3 | 9 |
| 2 | $\frac{1}{8}$ | $\frac{1}{24}$ | $\frac{1}{12}$ |
| 4 | $\frac{1}{4}$ | $\frac{1}{4}$ | 0 |
| 6 | $\frac{1}{8}$ | $\frac{1}{24}$ | $\frac{1}{12}$ |

- Find (i) $E(X)$ (ii) $E(Y)$ (iii) $E(XY)$ (iv) $\text{Cov}(X, Y)$
 (v) Marginal distribution of X and Y (06 Marks)
- b. Define (i) Null hypothesis (ii) Type-I and Type-II error (iii) Level of Significance (07 Marks)
- c. A sample of 100 tyres is taken from a lot. The mean life of tyres is found to be 40,650 kms with a standard deviation of 3260. Can it be considered as a true random sample from a population with mean life of 40,000 kms (use 0.05 level of significance). (Given $z_{0.05} = 1.96$, $z_{0.01} = 2.58$) (07 Marks)

OR

- 10 a. The joint probability distribution of two random variables X and Y are as follows:

| | | | | |
|-------|-----|-----|-----|-----|
| X \ Y | -2 | -1 | 4 | 5 |
| 1 | 0.1 | 0.2 | 0 | 0.3 |
| 2 | 0.2 | 0.1 | 0.1 | 0 |

- Determine : (i) Marginal distribution of X and Y (ii) Find $E(X)$, $E(Y)$ and $E(XY)$
 (iii) Covariance of X and Y (06 Marks)
- b. In the experiment of pea breeding the following frequencies of seeds were obtained.

| Round and Yellow | Wrinkled and Yellow | Rounded Green | Wrinkled and Green | Total |
|------------------|---------------------|---------------|--------------------|-------|
| 315 | 101 | 108 | 32 | 556 |

- Theory predicts that the frequencies should be in proportions 9:3:3:1. Examine the correspondence between theory and experiment. (Given $\chi_{0.05}^2 = 7.815$ for 3df). (07 Marks)
- c. A group of 10 boys fed on a diet A and another group of 8 boys fed on a different diet B for a period of 6 months recorded the following increase in weight (lbs).

| | | | | | | | | | | |
|----------|---|---|---|---|----|---|---|---|---|----|
| Diet A : | 5 | 6 | 8 | 1 | 12 | 4 | 3 | 9 | 6 | 10 |
| Diet B : | 2 | 3 | 6 | 8 | 10 | 1 | 2 | 8 | 5 | 5 |

- Test whether diets A and B differ significantly regarding their effect on increase in weight. (Given $t_{0.05}$ for 16 df = 2.12) (07 Marks)
